Least-squares best-fitting polynomials

1. Write down the system of linear equations that must be solved to find the least-squares best-fitting polynomial that finds the best linear and quadratic polynomials that pass through

(3.5, 4.7), (3.7, 5.2), (7.3, 6.3), (5.6, 5.7), (8.6, 7.9)

Answer: Note that the x values do not have to be sorted.

First, set $\mathbf{y} = \begin{pmatrix} 4.7 \\ 5.2 \\ 6.3 \\ 5.7 \\ 7.9 \end{pmatrix}$ and then $V_1 = \begin{pmatrix} 3.5 & 1 \\ 3.7 & 1 \\ 7.3 & 1 \\ 5.6 & 1 \\ 8.6 & 1 \end{pmatrix}$ and $V_2 = \begin{pmatrix} 12.25 & 3.5 & 1 \\ 13.69 & 3.7 & 1 \\ 53.29 & 7.3 & 1 \\ 31.36 & 5.6 & 1 \\ 73.96 & 8.6 & 1 \end{pmatrix}$.

Next, solve $V_1^{\mathrm{T}} V_1 \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} = V_1^{\mathrm{T}} \mathbf{y}$ and $V_2^{\mathrm{T}} V_2 \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = V_2^{\mathrm{T}} \mathbf{y}$. The solution to the first gives the coefficients for the least-squares best-fitting linear polynomial $a_1 t + a_0$ and the solution to the second gives the coefficients for the least-squares best-fitting quadratic polynomial $a_2 t^2 + a_1 t + a_0$.

2. Write down the system of linear equations that must be solved to find the least-squares best-fitting polynomial that finds the best linear and quadratic polynomials that pass through

(–2, 4.7), (–1, 5.2), (0, 5.7), (1, 6.3), (2, 7.9)

Answer:

First, set $\mathbf{y} = \begin{pmatrix} 4.7 \\ 5.2 \\ 5.7 \\ 6.3 \\ 7.9 \end{pmatrix}$ and then $V_1 = \begin{pmatrix} -2 & 1 \\ -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$ and $V_2 = \begin{pmatrix} 4 & -2 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix}$.

Next, solve $V_1^{\mathrm{T}} V_1 \begin{pmatrix} a_1 \\ a_0 \end{pmatrix} = V_1^{\mathrm{T}} \mathbf{y}$ and $V_2^{\mathrm{T}} V_2 \begin{pmatrix} a_2 \\ a_1 \\ a_0 \end{pmatrix} = V_2^{\mathrm{T}} \mathbf{y}$. The solution to the first gives the coefficients for the least-squares best-fitting linear polynomial $a_1 t + a_0$ and the solution to the second gives the coefficients for the least-squares best-fitting quadratic polynomial $a_2 t^2 + a_1 t + a_0$.

2. Suppose you find the least-squares best-fitting polynomial of degree $n + 1$ that passes through the $n + 1$ points $(x_0, y_0), \ldots, (x_n, y_n)$ where all the x values are different. Is this equal to the interpolating polynomial that passes through these points?

Answer: Yes. Because the interpolating polynomial passes through all the y values, so the error is zero.